

Analysis of nonlinear fractional Duffing oscillator using Homotopy analysis method (HAM)

Ejikeme C.L., Agbebaku D.F., Okofu M.B. and Oyesanya

Department of Mathematics University of Nigeria, Nsukka 410010, Enugu-State, Nigeria,
E-mail address: chioma.ejikeme@unn.edu.ng

Abstract

We investigated the analytical solutions of the forced Duffing oscillator with fractional order derivatives given by the equation

$$\begin{cases} D^\alpha u + \delta D^\beta u + \rho u + \mu u^3 = \lambda \sin \omega t, & t \in [0, \infty) \\ u(0) = a, \quad D^\beta u(0) = b, & a, b \neq 0, \text{ are real or complex constants} \end{cases} \quad (1)$$

where D denote the Caputo derivative, u is the displacement function, $\delta > 0$ is the damping factor, $m = [\alpha]$, $0 < \beta \leq 1$, $1 < \alpha \leq 2$, $\mu > 0$, $\rho > 0$, $\lambda > 0$ is amplitude of vibration, ω is the frequency.

We obtained the solution to the above equation by converting it to its augmented system with the same order and applied Homotopy Analysis Method (HAM) with a proper choice of the initial conditions. We observed that the solution of the fractional order case coincides with that of an integer case for $\alpha = 2$ and $\beta = 1$. Finally, the accuracy properties were demonstrated with an example.

Keywords: Fractional differential equations; Homotopy analysis method; Duffing Oscillator; Mittag-Leffler function.

Category: Mathematical physics.